Cavity Field Reconstruction at Finite Temperature

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Abstract

We present a scheme to reconstruct the quantum state of a field prepared inside a lossy cavity at finite temperature. Quantum coherences are normally destroyed by the interaction with an environment, but we show that it is possible to recover complete information about the initial state (before interaction with its environment), making possible to reconstruct any s-parametrized quasiprobability distribution, in particular, the Wigner function.

1 Introduction

Recently there have been proposals to reconstruct the quantum state of electromagnetic fields inside cavities [1, 2]. The reconstruction of non- classical states is a central topic in quantum optics and related fields, and there have been a number of proposals to achieve it (see for instance [3]). In fact, the full reconstruction of nonclassical field states [4] as well as of (motional) states of an ion [5] have been experimentally accomplished. The reconstruction is normally achieved through a finite set of either field homodyne measurements, or selective measurements of atomic states [1].

However, the presence of noise and dissipation has normally destructive effects. In fact, the reconstruction schemes themeselves also indicate loss of coherence in quantum systems [5]. Schemes for compensation of losses have already been proposed [6] and the relation between losses and s-parametrized quasiprobabilities has been pointed out in ref [7]. A method of reconstruction of the Wigner function that takes into account losses (at $T = 0$) has also been presented [2].

We consider here a single mode high-Q cavity where we suppose that a nonclassical field is prepared. The first step of our method consists in driving the generated state by a coherent pulse. The reconstruction of the field is done after turning-off the driving field, i.e. at a time when the cavity field has interacted with its environment at finite temperature. We show that by measuring the density matrix diagonal elements and properly weighting them, we can obtain directly the Wigner function even at $T \neq 0$. We should remark that to know a state, one has to have information about all the density matrix elements (diagonal and off-diagonal), however, with the method presented here (see also [2]), it is only necessary to have information about diagonal matrix elements.

2 Master equation and its solution

The master equation in the interaction picture for the reduced density operator $\hat{\rho}$ relative to a driven cavity mode, taking into account cavity losses at non-zero temperature and

under the Born-Markov approximation is given by [8] (in a frame rotating at the field frequency ω)

$$\frac{\partial\hat{\rho}}{\partial t} = (\hat{\mathcal{R}} + \hat{\mathcal{L}})\hat{\rho}, \tag{1}$$

where

$$\hat{\mathcal{L}}\hat{\rho} = (\hat{\mathcal{L}}_1 + \hat{\mathcal{L}}_2)\hat{\rho} \tag{2}$$

with

$$\hat{\mathcal{L}}_1\hat{\rho} = \frac{\gamma(\bar{n}+1)}{2}\left(2\hat{a}\hat{\rho}\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^\dagger\hat{a}\right), \qquad \hat{\mathcal{L}}_2\hat{\rho} = \frac{\gamma\bar{n}}{2}\left(2\hat{a}^\dagger\hat{\rho}\hat{a} - \hat{a}\hat{a}^\dagger\hat{\rho} - \hat{\rho}\hat{a}\hat{a}^\dagger\right), \tag{3}$$

and

$$\hat{\mathcal{R}}\hat{\rho} = -\frac{i}{\hbar}[\hat{H}, \hat{\rho}], \tag{4}$$

where

$$\hat{H} = i\hbar\left(\alpha^*\hat{a} - \alpha\hat{a}^\dagger\right). \tag{5}$$

$\hat{a}$ and $\hat{a}^\dagger$ are the annihilation and creation operators, γ the (cavity) decay constant, $\bar{n}$ is the mean number of thermal photons and α the amplitude of the driving field.

2.1 Displacing the field

The formal solution to Eq. (1) is given by (see for instance [9])

$$\hat{\rho}(t) = \exp\left[(\hat{\mathcal{R}} + \hat{\mathcal{L}})t\right]\hat{\rho}(0). \tag{6}$$

It is not difficult to show that Eq. (6) can be factorized in the product of two exponentials, one containing the reservoir (super) operators and the other the interaction (5), the latter one yielding an effective displacement on the initial field. In order to show this we calculate the commutator

$$[\hat{\mathcal{R}}, \hat{\mathcal{L}}]\hat{\rho} = \frac{\gamma}{2}\hat{\mathcal{R}}\hat{\rho}, \tag{7}$$

which allows the factorization

$$\hat{\rho}(t) = \exp(\hat{\mathcal{L}}t)\exp\left[-\frac{2\hat{\mathcal{R}}}{\gamma}(1-e^{\gamma t/2})\right]\hat{\rho}(0). \tag{8}$$

After driving the initial field during a time t, the resulting field density operator will read

$$\hat{\rho}(t) = e^{\hat{\mathcal{L}}t}\hat{\rho}_\beta(0), \tag{9}$$

where

$$\hat{\rho}_\beta(0) = \hat{D}^\dagger(\beta)\hat{\rho}(0)\hat{D}(\beta), \tag{10}$$

with the effective (displacing) amplitude

$$\beta = -2\alpha\frac{1-e^{\gamma t/2}}{\gamma}. \tag{11}$$

2.2 Solution to the Master Equation at finite temperature

We now obtain the density matrix (4), by defining [9]

$$\hat{J}_-\hat{\rho} = \hat{a}\hat{\rho}\hat{a}^\dagger, \qquad \hat{J}_+\hat{\rho} = \hat{a}^\dagger\hat{\rho}\hat{a}, \qquad \hat{J}_3\hat{\rho} = \hat{a}^\dagger\hat{a}\hat{\rho} + \hat{\rho}\hat{a}^\dagger\hat{a} + \hat{\rho}, \tag{12}$$

where the superoperators $\hat{J}_-$, $\hat{J}_+$ and $\hat{J}_3$ obey the commutation relations $[\hat{J}_-, \hat{J}_+]\hat{\rho} = \hat{J}_3\hat{\rho}$ and $[\hat{J}_3, \hat{J}_\pm]\hat{\rho} = \pm 2\hat{J}_\pm\hat{\rho}$, which can be written as

$$\hat{\rho}(t) = e^{\frac{\gamma t}{2}} e^{\Gamma_{\bar{n}}(t)\hat{J}_+}\left[\frac{e^{-\gamma t/2}}{1+N_t}\right]^{\hat{J}_3} e^{\Gamma_{\bar{n}+1}(t)\hat{J}_-}\hat{\rho}_\beta(0), \tag{13}$$

and we have defined

$$\Gamma_{\bar{n}}(t) = \frac{\bar{n}(1-e^{-\gamma t})}{1+N_t}, \qquad \Gamma_{\bar{n}+1}(t) = \frac{(\bar{n}+1)(1-e^{-\gamma t})}{1+N_t}, \tag{14}$$

with $N_t = \bar{n}(1-e^{-\gamma t})$.

3 The reconstruction method

We now calculate the diagonal density matrix elements $< m|\hat{\rho}(t)|m >$ from (13)

$$\langle m|\hat{\rho}_\beta(t)|m\rangle = \frac{1}{1+N_t}\sum_{k=0}^{\infty}\sum_{n=0}^{\infty}\begin{pmatrix} k \\ n \end{pmatrix}\begin{pmatrix} m \\ n \end{pmatrix}[\Gamma_{\bar{n}}(t)]^{m-n}[\Gamma_{\bar{n}+1}(t)]^{k-n}\frac{e^{-n\gamma t}}{[1+N_t]^{2n}}\langle k|\hat{\rho}_\beta(0)|k\rangle. \tag{15}$$

Multipliying the expression above by powers of the weight function χ_s, where

$$\chi_s = \frac{\frac{s+1}{s-1} - \Gamma_{\bar{n}+1}(t)}{\frac{e^{-\gamma t}}{[1+N_t]^2} + \Gamma_{\bar{n}}(t)\left(\frac{s+1}{s-1} - \Gamma_{\bar{n}+1}(t)\right)} \tag{16}$$

and adding over m we obtain

$$F(\beta; s) = \sum_{m=0}^{\infty}\chi_s^m\langle m|\hat{\rho}_\beta(t)|m\rangle = \frac{1}{[1+N_t][1-\chi_s\Gamma_{\bar{n}}(t)]}\sum_{k=0}^{\infty}\left(\frac{s+1}{s-1}\right)^k\langle k|\hat{\rho}_\beta(0)|k\rangle, \tag{17}$$

where we have used the fact that [10]

$$\sum_{m=0}^{\infty}\begin{pmatrix} m \\ n \end{pmatrix}x^m = \frac{x^n}{[1-x]^{n+1}}. \tag{18}$$

If we now multyply $F(\beta; s)$ by the quantity

$$-\frac{2[1+N_t][1-\chi_s\Gamma_{\bar{n}}(t)]}{\pi(s-1)}, \tag{19}$$

we finally obtain

$$W(\beta; s) = -\frac{2}{\pi(s-1)}\sum_{k=0}^{\infty}\left(\frac{s+1}{s-1}\right)^k\langle k|\hat{\rho}_\beta(0)|k\rangle, \tag{20}$$

which is the s-parametrized quasiprobability distribution [11].

Therefore, by measuring the diagonal elements of the evolved field density matrix, Eq. (15), we may obtain complete information on the initial state.

We should remark that in this case (thermal environment), more information on the $P_m(t) =< m|\hat{\rho}(t)|m >$ is required than in the zero temperature case. Nevertheless, it is possible to find a weight function that allows the reconstruction of the initial field.

3.1 Measuring the photon distribution

For completness, we suggest a way to measure the photon number distribution $P_m(\beta,t)$ of Eq. (17). It is not difficult to show that the atomic inversion for the case of a three-level atom in a cascade configuration with the upper and the lower levels having the same parity and satisfaying the two-photon resonance condition is given by (see for instance [12])

$$W(\beta;t+\tau)\cong\sum_{m=0}^{\infty}P_m(\beta,t)\cos([2m+3]\lambda\tau), \tag{21}$$

where λ is the atom-field coupling constant. In order to obtain P_m from a family of measured population inversions, we invert the Fourier series in Eq. (21), or

$$P_m(\beta,t)=\frac{2\lambda}{\pi}\int_0^{\tau_{max}}d\tau W(t+\tau)\cos([2m+3]\lambda\tau). \tag{22}$$

We need a maximum interaction time $\tau_{max}=\pi/\lambda$ much shorter than the cavity decay time, which implies we must be in the strong-coupling regime, i.e. $\lambda >> \gamma$.

4 Conclusions

We have presented a method to reconstruct the Wigner function (and in general any quasiprobability distribution) of an initial nonclassical state at times when the field would have normally lost its quantum coherence because of the interaction with an environment at finite temperature. This is an extension of our previous work [2] where we considered the interaction with the environment at zero temperature. The crucial point of our approach is the apropriate weighting of the evolved (driven and decayed) photon number distribution. Driving the initial field immediately after preparation, is not only useful for covering a region in phase space but also makes possible, together with the weight function, χ_s, to store quantum coherences in the diagonal elements of the time evolved density matrix. We have shown here that it is possible to find a weight function that allows to reconstruct the initial field even at finite temperature, and this is the main result of our paper.

Similar conclusions may be reached by employing the method of generating functions [13]. The possibility of reconstructing quantum states when the system interacts with an environment maybe relevant for applications in quantum computing. Loss of coherence for such interactions is likely to occur in such devices, and our method could be used, for instance, as a scheme to refresh the state of a quantum computer [14] in order to minimize the destructive action of a hot environment.

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